

MATH 456 (Elementary Real Analysis) – Spring 2015
Dr. Melvin Royer

Description:	A theoretical study of single variable calculus. Topics include the real number system, sequences, limits, continuity, derivatives, Riemann integrals, and convergence of series. Emphasis will be placed on understanding, writing, and communicating proofs.					
Objectives:	<ol style="list-style-type: none"> 1. Demonstrate understanding of the rigorous foundations of calculus at a level sufficient for success in graduate study 2. Write correct and clear mathematical analysis proofs 3. Explain the implications of the completeness of the real numbers 4. Consistently apply the concept of mathematical limit to problems across the curriculum of this first course in analysis 					
Prerequisite:	Calculus III (MAT 255 or equivalent)					
Materials:	<ol style="list-style-type: none"> 1. Text: <u>Analysis with an Introduction to Proof, 5th Edition</u>; Lay; Prentice Hall; 2014 2. Graphing calculator or laptop with Maple 3. 3-ring binder with extra filler paper 					
Instructor Contacts:	<ol style="list-style-type: none"> 1. Email: melvin.royer@indwes.edu 2. Phone: 677-2987 (Office), 662-1673 (Home, before 9:30 PM) 3. FAX: 677-1704 4. Office: Ott Hall Science/Nursing, Room 170-D 					
Listed Office Hours: Also, by appointment or walk-in	Times	Monday	Tuesday	Wednesday	Thursday	Friday
	7:50		MAT-456		MAT-456	
	8:55	Office	MAT-456	Office	MAT-456	Office
	10:00	Chapel	Office	Chapel	Office	Chapel
	11:15	MAT-204		MAT-204		MAT-204
	12:20	Office		Office		Office
	1:25		Office		Office	
	2:30					
3:35						

Tentative Schedule

Date	Day	Topics	Homework	Boardwork
Jan 6	T	Review, ordered fields	<i>Read Sections 1.1 – 3.1</i> 3.2 – 3(ejm),4,6(a),7,10	
Jan 8	R	Completeness	3.3 – 1,2,3,4,7,8,10	
Jan 13	T	Topology	3.4 – 1,2,3,4,5,6,12,14,15,17,21	3.3-8 Chris
Jan 15	R	Compactness	3.5 – 1,2,3,4,5,8,12	3.4-14 Lauren
Jan 20	T	Convergence of sequences	4.1 – 1,2,6(cd),7(ef),9,10,11,13,15,16(ab)	3.5-8 Luke
Jan 22	R	Limit theorems	4.2 – 1,2,4,5(cij),7,8,9,12,15,17,18	4.1-15(b) Josh
Jan 27	T	Monotone & Cauchy sequences	4.3 – 1,2,3(bd),4, 9,11,13,15	4.2-12 Chelsea
Jan 29	R	Subsequences	4.4 – 1,2,4,9,10,13,14,15,17,18	4.3-13 Caryn
Feb 3	T	Limits of functions	5.1 – 1,2,3(fh),6(a),7(b),9,10(a),13,14,15,16,18	4.4-15 Adam
Feb 5	R	TEST 1 (Ch 3, 4)		
Feb 10	T	Continuity	5.2 – 1,2,5,7,8,10(a),11,13,14,17,18	5.1-16 Josh
Feb 12	R	Properties of continuous functions	5.3 – 1,2,3(ab),5,6,7,13,14,15	5.2-18 Caryn
Feb 17	T	Uniform continuity	5.4 – 1,2,3,4(b),6,8,10,11	5.3-13 Adam
Feb 19	R	Derivatives	6.1 – 1,2,4(e),6,7,8,9,11,17	5.4-8 Chris
Feb 24	T	Mean Value Theorem	6.2 – 1,2,5(af),8,9,11,15,20,23	6.1-11 Chelsea
Feb 26	R	Taylor's Theorem	6.4 – 1,2,4,5,7,9,10,11	6.2-15 Lauren
Mar 3	T	SPRING BREAK		
Mar 5	R	SPRING BREAK		
Mar 10	T	Riemann integral	7.1 – 1,2,4,5,11,13,14,16,17	6.4-11 Luke
Mar 12	R	TEST 2 (Ch 5, 6)		
Mar 17	T	Properties of the Riemann integral	7.2 – 1,2,4,5,6,9,10,11,12,14,17	7.1-16 Adam
Mar 19	R	Fundamental Theorem of Calculus	7.3 – 1,2,5(d),8,10,12,13,17(a)	7.2-14 Chelsea
Mar 24	T	Infinite series Convergence tests	8.1 – 1,2,4(c),5(df),8,9,13 8.2 – 1,2,3(ejo)	7.3-12 Luke
Mar 26	R	Convergence tests Power series	8.2 – 4(a),5(ei),6,8,11,12 8.3 – 1,2,3(h),4,5(bc),10	8.1-8 Caryn
Mar 31	T	Pointwise and uniform convergence	9.1 – 1,2,3,4,8,10,15	8.2-8 Lauren
Apr 2	R	TEST 3 (Ch 7, 8)		
Apr 7	T	Convergence	9.2 – 1,2,3,5,6,11,12,13	9.1-8 Chris
Apr 9	R	Convergence of power series	9.3 – 1,2,3,7,10,11,13	9.2-6 Josh
Apr 14	T	EARLY FRIDAY		
Apr 16	R	STUDY DAY		

Final Exam: Monday April 20, 8:00 – 9:50 a.m.

Advice & Encouragement:

1. “To every thing there is a season, and a time to every purpose under the heaven . . . a time to keep silent and a time to speak . . .” Eccl 3:1, 7. Usually in a college math course, there is more problem with students keeping silent than with them speaking. Questions are appropriate and appreciated at any time. Giving answers to my questions is also appreciated (unless you just answered the last question; then giving another student the opportunity to answer would be appropriate.)
2. “Consider the path for your feet and let all your ways be established.” Prov 4:26. The schedule shows what we will be covering the next class period. The wise will have previewed the section(s) before coming to class; all the relevant studies in learning show that having some prior knowledge greatly increases the rate and level of comprehension.

Physical preparedness is also important. Your mind resides in a body that was created to sleep, eat nutritious meals, etc. If you are constantly only one step ahead of your next deadline, please consult with your advisor and/or a trusted friend about simplifying your life – a good college experience requires spiritual and academic reflection time.

3. “. . . Talk about [these commandments] when you sit at home and when you walk along the road, when you lie down and when you get up. Tie them as symbols on your hands and bind them on your foreheads. Write them on the doorframes of your houses and on your gates.” Deut 6:7-9. There is no substitute for consistently doing the homework. Mathematics has many characteristics of a foreign language. Your retention will be much better if you study every day than if you wait until the weekend or just before the test to try to catch up.
4. “As iron sharpens iron, so one man sharpens another.” Prov 27:17. My observation has been that students perform better (often much better) when they make a serious effort to become part of the class “community.” Academic benefits of studying and socializing together include the following: extra feedback to get your misconceptions corrected, other perspectives on what the important issues really are, emotional energy when preparing for tests, an emergency contact when you forget what assignment is due. . .
5. “And unto one he gave five talents, to another two, and to another one. . .” Matt 25:15. There will probably be a wide range of backgrounds and abilities in the class. Be respectful of others. Compete against your own God-given ability, not against each other. Don’t be too embarrassed to ask “stupid” questions.
6. “Therefore, since I myself have carefully investigated everything from the beginning, it seemed good also to me to write an orderly account for you . . .” Luke 1:3 Don’t wait until the exam to try to write an orderly account of what you think you know – you need prior feedback. If you don’t understand a homework or practice test problem, unless I specifically say so, it would be less than wise to ignore it and hope it goes away. Your professors have office hours for a reason, but it must be you who makes the effort to make contact.
7. “For God so loved the world that He gave His one and only Son, that whoever believes in Him shall not perish but have eternal life.” John 3:16. Work hard and take the course seriously, but don’t neglect your spiritual life. Ultimately, the only mathematics you need to know is that God has only one Son, and that there are only two places to spend eternity.

Grading:

1. *Homework*

Homework will be assigned almost every day; unless otherwise stated, my assumption is that you will have it completed by the next class meeting. Working together on the homework is strongly encouraged. Homework for the week will normally be collected once per week. Grading will be based partly on quantity (percentage of assignment completed) and partly on quality (a few randomly chosen problems graded for correctness). Answers are not sufficient; work must be shown on all problems which cannot be done entirely mentally.

All homeworks will be worth the same number of points. Your lowest homework score will be dropped and your percentage on the remaining scores will be normalized to 200 points. Assignments must be submitted on the due date. No late homework will be accepted. Exceptions to this policy will be made only upon submission of official university documentation of an illness or university sponsored function. Dropping your lowest score is my way of accounting for the fact that everyone has emergencies; do not waste your drop opportunity for preventable reasons. If you know you must miss a class, make arrangements with me ahead of time to turn in the homework early.

2. *Presentations*

Students will be individually pre-assigned selected homework problems to present to the class on the board. (You may project pre-drawn figures on the screen if you wish, but the actual work should be done on the board). These presentations will occur at the beginning of most class periods. Most of the selected problems are proofs; the general rule is that you may only use theorems/examples/exercises in our textbook that occur BEFORE the problem you are proving.

3. *Participation*

Worksheets and other classwork will combine with attendance to count for a participation grade.

4. *Tests*

Tests will be given in class near the dates indicated on the schedule. (The exact date may be slightly different than indicated, but will be announced at least one week in advance). If you must miss a test and provide me with a legitimate reason in advance, you may take a makeup test without penalty. If you do not give advanced notice, you may not be allowed to take a makeup at all, or points may be deducted from your test score as a penalty. In case of a medical or family emergency, please notify me immediately by any of the contact means listed above. My willingness to accommodate your emergency will be related to your prompt efforts to notify and communicate with me.

Some form of technology (a laptop computer with Maple or a graphing calculator) is required for all tests.

5. *Final Exam*

The final exam will be comprehensive. If it is impossible for you to take the final exam at the scheduled time, prior arrangements must be made to take the exam at a later time (which must be later than the scheduled time). If your percentage on the final exam is higher than your lowest test score, I will replace that test score with your final exam percentage.

6. *Professional Development*

Participation in any of the following out-of-class activities will add the indicated number of points to the student's point total as well as his/her points possible. For example, suppose Students #1 and #2 each earn 650 total class points, but Student #2 also earns 50 professional development points. Student #1 will then have a course average of $650/800 = 81.2\%$, while student #2 will have an average of $700/850 = 82.4\%$. A maximum of 100 professional development points is allowed.

Students may propose math-related activities other than the following for instructor approval.

<i>Activity</i>	<i>Points</i>
IWU MAC Meeting	10
IWU Math Department public relations event (attending/presenting)	10/20
IWU Math Colloquium (attending/presenting)	10/20
Math contests	Up to 25
Off-campus math conference (pre-approved by instructor)	Up to 50

7. *Weighting*

3 Tests	300 pts	38%
Homework	200 pts	25%
Presentations	50 pts	6%
Participation	50 pts	6%
Comprehensive Final Exam	200 pts (see above replacement policy)	25%
TOTAL	750 points	100%

8. *Scale*

The following minimum percentages will guarantee the indicated grade. At the end of the course, the scale may be curved to be more lenient, but do not count on this.

Grade	A	A-	B+	B	B-	C+	C	C-	D+	D
Guaranteed Percentage	93	90	87	83	80	77	73	70	67	60

Citizenship:*Attendance:*

Enrolled students are responsible for attending all sessions of this class. Attendance may be taken without notice on any given class day and used as a homework score. Poor attendance has always been a leading cause of failure in math courses. Our mutual goal is your success in this course.

Distractions:

Tardiness is an unacceptable distraction to the other members of the class. Everyone occasionally runs a few minutes behind, but being tardy an excessive amount or an excessive number of times may result in denial of credit for work due that week.

Usage of cell phones, MP3 players, laptops (other than for note-taking or classwork), etc., during class is inappropriate and discourteous to other members of the class.

Email: If necessary, email will be sent to students' IWU student email accounts. Such email is considered official university correspondence and students are responsible for checking their email on a daily basis.

Topics List

- 1) Properties of the Real Numbers
 - a) Ordered Fields
 - b) Bounds and Completeness
 - c) Topological Properties
 - i) Boundary and Accumulation Points
 - ii) Bolzano-Weierstrass Theorem
 - iii) Compactness and Heine-Borel Theorem
- 2) Sequences
 - a) Convergence
 - b) Special types (bounded, monotonic, Cauchy)
 - c) Subsequences
- 3) Continuity
 - a) Limits
 - b) Properties of Continuous Functions
 - c) Intermediate Value Theorem
 - d) Uniform Continuity
- 4) Differentiation
 - a) Derivatives
 - b) Mean Value Theorem
 - c) Taylor's Theorem
- 5) Integration
 - a) Riemann Integral
 - b) Fundamental Theorem of Calculus
- 6) Infinite Series
 - a) Convergence Tests
 - b) Convergence of Power Series
- 7) Sequences and Series of Functions
 - a) Pointwise and Uniform Convergence
 - b) Weierstrass M-test
 - c) Limits of Uniformly Convergent Functions

Mathematics Department Mission Statement and Program Objectives

“The Department of Mathematics is committed to preparing students academically and spiritually for careers and/or advanced study. This preparation will occur through a Christian liberal arts curriculum developing solid foundations in mathematics content, technological skills, and critical analysis and problem solving ability. Communication, teamwork, and leadership skills will be developed through a multi-disciplinary philosophical approach in integrating faith, learning, and service.”

- 1) PROOF: ...understand and proficiently implement the logical role and methodology of rigorous proof in the axiomatic development of mathematics.
- 2) CONTENT: ...understand core mathematical content in standard areas of analysis, algebra, geometry, and probability.
- 3) NATURE: ...proficiently articulate the overall nature of mathematics including its history; current organization; and interfaces with Christian faith, philosophy, and other academic disciplines.
- 4) APPLICATION: ...proficiently analyze, model, and solve real-world problems using mathematical/statistical software as appropriate.
- 5) COMMUNICATION: ...proficiently communicate mathematics in both written and oral form

MAT-456 primarily addresses objectives #1, #2, and #5.